APPLICATIONS OF AMPERE’S CIRCUITAL LAW

(i) Magnetic field induction due to a current carrying straight conductor

Consider a point P at a distance R from the straight conductor.

By symmetry all points at distance r will be on a circle of radius R.

Using Ampere's circuital law for P

\[ \int B \cdot dl = \mu_0 I \]

\[ B \int dl = \mu_0 I \]

\[ B \cdot 2\pi R = \mu_0 I \]

\[ \Rightarrow B = \frac{\mu_0 I}{2\pi R} \]

(ii) Magnetic field inside an infinitely long current carrying solenoid

If there is a long solenoid of length l as shown.

Experimentally it has been observed that magnetic field outside is very small compared with the field inside.

Applying Ampere's circuital law

\[ \int B \cdot dl = \mu_0 i \]

\[ \int B \cdot dl = \int_a^b B \cdot dl + \int_c^d B \cdot dl + \int_b^c B \cdot dl + \int_d^a B \cdot dl \]

here \[ \int_b^c B \cdot dl = \int_d^a B \cdot dl = 0 \]

as both are perpendicular to field lines.
and \[ \oint \mathbf{B} \cdot d\mathbf{l} = 0 \] as \( B = 0 \) outside the solenoid.

so \[ \oint \mathbf{B} \cdot d\mathbf{l} = \int_{a}^{b} \mathbf{B} \cdot d\mathbf{l} = B \int_{a}^{b} dl = Bx \] \( \text{(i)} \)

if there are \( n \) turns per unit length with \( i_0 \) current in each turn then within length of \( x \), net current enclosed

\[ i = nx_{0} \] \( \text{(ii)} \)

form (i) & (ii)

\[ Bx = \mu_{0} nx_{0} \]

or

\[ B = \mu_{0} n i_{0} \] \( \text{(iii)} \)

if the solenoid is wrapped on a core of permeability \( \mu_{m} \) then

\[ B = \mu_{m} n i_{0} = \mu_{0} \mu_{m} n i_{0} \] \( \text{(iv)} \)

(iii) **Magnetic field of induction due to a current carrying cylinder**

Consider a cylinder of radius \( R \) with current \( I \) passing through it.

![Diagram of cylinder with magnetic field](image)

Magnetic field at any point at \( r \) distance form the axis of cylinder where \( r > R \) will be

\[ \oint \mathbf{B} \cdot d\mathbf{l} = B \oint dl = B2\pi r \] \( \text{(i)} \)

as \( r > R \) current will pass through the circuit of radius \( r \). so

\[ B_{z} \pi r = \mu_{0} I \]
\[ B = \frac{\mu_0 I}{2\pi r} \]  

...(ii)

It is same as for an infinite line current element.

**When \( r < R \)**

Consider a Gaussian surface of radius \( r \) inside the cylinder and current enclosed by the inner circle of radius \( r \) is given by

\[
I' = \frac{\text{Current}}{\text{Area of the inner circle}} \times \text{area of the inner circle}
\]

\[
I' = \frac{I}{\pi R^2} \times \pi r^2 = \frac{Ir^2}{R^2}
\]

...(iii)

so

\[
B_2 r = \mu_0 I' = \mu_0 \frac{Ir^2}{R^2}
\]

so

\[
B = \frac{\mu_0 Ir}{2\pi R^2} \text{ for } r < R
\]

...(iv)

**Inside a hollow cylinder**

When there is a hollow cylinder, then whole current will exist only on the surface of the cylinder and inside current will become zero, so \( I = 0 \)

hence \( B 2\pi r = 0 \)

or \( B = 0 \)